Numbers

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1 Objectives

- In this chapter you'll be able to work with
  - real number systems.
  - subsets of real numbers.
  - Arithmetic operations on and with integers, square roots, and irrational numbers.

2 Real Numbers

Real numbers can be classified into four categories:

- Rational Numbers

  Are any numbers which can be put in the form of

  $$\frac{a}{b}$$

  where $b \neq 0$.

- Irrational Numbers

  Are any non-repeating or non-terminating numbers which can not be put in the form of

  $$\frac{a}{b}$$

  where $b \neq 0$.

- Natural numbers

  Set of *Positive whole numbers* starting from 1 and increasing:

  1,2,3,4,......

- Integers

Set of *positive and negative* whole numbers, including zero.

-5,-4,0, 1,2,3,4,......

Quick overview of real numbers:

Classify the following in as many categories of real numbers as you can:

1. -4
2. 6
3. $\frac{-1}{2}$
4. -4
5. $\pi\sqrt{2}$

3 Squares and Square roots:

- The square root of a number is the value that, when multiplied by itself, gives the original number. i.e

$$4 = 2 \times 2.$$

  In this case, two is a square root of four, whereas four is the square of two.

  Example:

- A fence encloses a square skating rink of area, $36m^2$.

  How long is each side of the rink?

- **Sol:**

  Since the rink is in a square shape, then its area is given by the following formula.

$$Area = 36m^2 = side \times side.$$

  That is, $36m^2 = 6m \times 6m$. Therefore, 6 is the square root of 36.

  However, if we talk about absolute values or all possible answers, then$-6 \times -6 = 36 = (6)(6)$.

  So, note that, **Square Root of any number has two answers, except for time, distance and money or more, do some researching if in doubt.**

3.1 Type of Squares:

- **Perfect Square.**

  Is a number, say 49, which has a whole number as its principal square root.

  Since $7 \times 7 = 49$, then $\sqrt{49} = 7$

- **Imperfect Square:**

A number, say 4.41, which has a fractional root number, thus imperfect.

Find the square root in fraction form.

NB: To find the square root of a mixed fraction, start by changing it to improper form and then take the square root of the numerator and denominator separately.

1. $\sqrt{4.41}$

   Sol:

   Since

   $$2.1 \times 2.1 = 4.41$$

   , then

   $$\sqrt{\frac{441}{100}} = \sqrt{\frac{21 \times 21}{10 \times 10}} = \frac{21}{10}$$

2. $\sqrt{1\frac{9}{16}}$

   Sol:

   $$\sqrt{1\frac{9}{16}} = \sqrt{1\frac{16 \times 1 + 9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

- **Examples**

1. What is the square root of 9?

   Sol:

   Since $3 \times 3 = 9$, then $\sqrt{9} = 3$

2. At the Olympic Games in Barcelona, Spain, Jeff Thue from Port Moody, B.C., won silver medal in the Supper Heavyweight division of the wrestling competition.

   If the wrestling mat was in shape of a square with an area of $64m^2$, then what was the length of each side?

   Sol:

   Since $8 \times 8 = 64$, then $\sqrt{64} = 8$.

   Therefore, the length of a square mat whose $64m^2$ would be $\underline{8m}$.

- **Assignment**

1. Evaluate following, and then round your answers to the nearest tenth.
   (a) $\sqrt{121}$
   (b) $\sqrt{73}$
   (c) $\sqrt{4.84}$
   (d) $\sqrt{0.8}$

2. **Word Problems**

   (a) Prince Edward Island is the Canadian smallest province, with an area of $5660km^2$.
   (b) Canadian largest province, Quebec, has an area of $1540680km^2$

   If the two province were in the square shapes, what would be the length of each side of each province, to the nearest whole number, in kilometres?
   (c) About how many square Prince Edward Islands could fit into a square Quebec?
   (d) A square has an area of $81cm^2$.

   What's the length of one side, and the radius of the largest circle you can fit inside it?
   (e) For each of the following, explain if each statement is always true, sometimes true, or never true?

      i. The square of an even number is larger than the number.
      ii. The square of an even number is even.
      iii. The principal square root of a number is half the number.
      iv. The principal square root of a perfect square is less than 1.

4 Powers and Exponential forms

All organisms, regardless of how complicated they are, begin with a single cell. This cell splits to form 2 new cells. The new 2 cells split and the process continues.

Cell, powers and exponents			
Expression form	Factored form	Power of base 2	Exponent
2	2×1	2	1
$2^1 \times 2^1$	2×2	2^2	2
$2^2 \times 2^1$	$2 \times 2 \times \times 2$	2^3	3
$2^2 \times 2^2$	$2 \times 2 \times 2 \times 2$	2^4	4

1. Simplify and Evaluate the following
   (a) $3^4 \times 3^2$

   Sol:

Since we've the same base, 3, then we just add the exponents,

$$(4 + 2) = 6$$

Therefore,

$$3^4 \times 3^2 = 3^6 = 729$$

.

Power $= 3^6$ **whereas exponent** $= 6$

(b) $3^5 \times 3^3$

(c) $(2^3)^2$

Sol:

Since we've the same base, 2, then we multiple just the power/exponents,

$$(3 \times 2) = 6$$

Therefore,

$$(2^3)^2 = 2^6 = 64$$

.

(d) $4^4 \times 2^3$

4.1 Exponent Laws

In this section, we'll learn how to perform operations on powers. i.e

$$3^4 = 3 \times 3 \times 3 \times 3$$

4.1.1 Multiplying powers

Consider the following power products:

$$4^3 \times 4^2 = (4)(4)(4) \times (4)(4) = (4)(4)(4)(4)(4) = 4^5$$

That is to say, given the same base, 4, the result is the same base raised to the sum of the exponents/powers that were multiplied.

That's **Keep the base and add the exponents!**

$$(a^x)(a^y) = a^{x+y}$$

Remember

$$a = a^1$$

Examples:

1. $$625 \times 25 = 5^4 \times 5^2 = 5^{4+2} = 5^6$$

2. $$3^7 \times 3^{-2} = 3^{7+(-2)} = 3^5$$

4.1.2 Powers Raised to an exponent

Consider the following:

$$(5^3)^2 = (5^3)(5^3) = 5^{3\times2} = 5^6$$

From above, you should have noticed that the result, given the same base, is raised to the product of the two exponents.

That's

$$(a^x)^y = a^{xy}$$

Examples:

1. $$(6^2)^4 = 6^{2\times4} = 6^8$$

2. $$((-7)^2)^5 = (-7)^{2\times5} = (-7)^{10} = (-1)^{10} \times (7)^{10} = 7^{10}$$

NB: $(-1)^{evenexponent} = 1$ **and** $(-1)^{oddexponent} = -1$

4.1.3 Dividing Powers

Consider the following:

$$4^3 \div 4^2 = \frac{(4)(4)(4)}{(4)(4)} = 4$$

As we've already seen, dividing powers is a subtraction

$$(-3^5) \div (-3)^3 = \frac{(-3)(-3)(-3)(-3)(-3}{(-3)(-3)(-3)} = (-3)^2 = 9$$

keep the base and subtract the exponents.That's

$$a^x \div a^y = a^{x-y}$$

Or

$$\frac{a^x}{a^y} = a^{x-y}$$

NB: The exponent of the result is always the exponent of the <u>numerator minus the exponent of the denominator.</u>

Examples:

1.

$$8^{10} \div 8^2 = 8^{10-2} = 8^8$$

2.

$$\frac{5^6}{5^2} = 5^{6-2} = 5^4$$

3.

$$\frac{3^7}{4} \div \frac{3^3}{4} = 3^4$$

4.

$$5^6 \div 5^{-2} = 5^{6--2} = 5^{5^8}$$

NB: - - = +, - + = -, and + + = +

4.1.4 Zero and negative Exponents

Consider the following examples,

1.

$$4^3 \times 4^3 = 1 = 4^0$$

2.

$$(2x + y)^0 = 1$$

3.

$$4(3)^0 = 4 \times 1 = 4$$

Any base to the power of zero equals one!!

Now and again, consider the following:

1.

$$10^3 \div 10^5 = \frac{1}{10^2} = 10^{-2}$$

Notice what happens when we've a negative exponent. The result is the reciprocal of the base, with the exponent now positive. That's

$$a^{-x} = \frac{1}{a^x}$$

Or

$$a^x = \frac{1}{a^{-x}}$$

2.

$$(10)^5 = \frac{1}{10^{-5}}$$

3.

$$4^{-10} = \frac{1}{4^{10}}$$

4.

$$[\frac{3}{4}]^{-2} = \frac{4}{3}$$

5.

$$\frac{x}{5}^{3} = \frac{x^3}{125}$$

Always remember that the denominator of a fraction cannot be zero, otherwise, it's undefined

Key point:

- Make sure that you are comfortable with rational and irrational numbers, square roots, powers and the exponent laws. More,Watch all the videos and ask questions if you are in doubt.

  Assignment

  1. $$7^4 \times 7^{-5} \times 7^2$$

  2. $$\frac{y}{3}$$

  3. $$\frac{120 \times x^{-4} \times y}{6 \times x^2 \times y^-5}$$

  4. $$(\frac{14x}{18y^2})^0$$

  5. $$(\frac{4x^2y}{y^{-5}})^{-2})^2$$

- **Word Problems**

  1. An electron has a mass of 9.1×10^{-28} grams.

     How many electrons have a mass of 1 g?

  2. A proton has a mass 1800 times that of an electron.

     How many protons have a mass of 2g?

4.2 Scientific Notation

Scientists often deal with large numbers. To make notation easier, they use a short form that involves decimals and powers of ten.

This short form is called **Scientific Notation.**

There are two types of scientific notation as listed below.

1. **Large numbers**

   The following are key points to remember when writing/dealing with scientific notations of large numbers.

(a) How many digits are written to the left of the decimal points.

(b) What determines the value of the exponent in the power of 10?

(a) **Examples:**

- Write each number in scientific notation.

i. 3250000

Sol:

$$3250000 = 3.25 \times 10^6$$

ii. 4000000

Sol:

$$40000000 = 4.0 \times 10^7$$

iii. 0.00000001

Sol:

$$1.0 \times 10^{-8}$$

(b) **Assignment**

i. 150000

ii. 7.96×10^{-7}

iii. 75

Word Problems

Express each of the numbers in scientific notation.

iv. The area of Manitoba is about $650000km^2$.

v. The Jurassic period lasted from about 210000000 to 140000000

vi. The length of Yukon Territory coastline is 343 km.

A. Write this number in scientific notation

B. Express the length of the Yukon Territory coastline in metres. Write the result in scientific notation.

2. **Small Numbers**

Scientific notation is used as a short form for small numbers as well as for large numbers. in this section, we're going to study the small numbers written in standard form and in scientific notation.

(a) Key points to remember when dealing with small numbers:
   - What determines the value of the exponent in the power of the 10?
   - A rule for expressing small numbers in scientific notation.

(b) **Examples:**

   i. The mass of an electron is about 0.00054 of the mass of a proton.

   Sol:

$$0.00054 = 5.4 \times 10^{-4}$$

   ii. 0.0000003

   Sol:

$$0.0000003 = 3.0 \times 10^{-7}$$

(c) **Assignment**

Write each of the following in scientific notation/form.

   i. 0.00000027
   ii. 0.00000010
   iii. 0.0000001
   iv. 0.000000

   Word Problems

   v. The diameter of some cells is 0.005
   vi. The percent of neon in the dry air at sea level is about 0.0018.
   vii. Victoria gets 0.37 times as much as much snow fall as Edmonton

NB: Videos, PQ, and answers to be separately posted

[1] [3] [2]

References

[1] B.C. : Academic Edge Brent wood Bay. Math grade 9: Go beyond your limits. 1995.

[2] Mun Castle Rock Research Corp. The key study guide: Math 9. 2003.

[3] Pearson. Lorraine. Math grade 9: Makes sense. 2009.